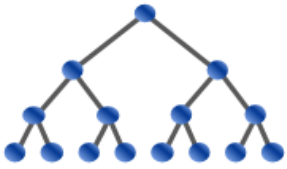

USACO 2024 JANUARY CONTEST, PLATINUM
PROBLEM 3. MOOBALL TEAMS III

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Contest has ended.

Log in to allow submissions in analysis mode

English (en)

Farmer John has N cows on his farm ($2 \leq N \leq 2 \cdot 10^5$), conveniently numbered $1 \dots N$. Cow i is located at integer coordinates (x_i, y_i) ($1 \leq x_i, y_i \leq N$). Farmer John wants to pick two teams for a game of mooball!

One of the teams will be the "red" team; the other team will be the "blue" team. There are only a few requirements for the teams. Neither team can be empty, and each of the N cows must be on at most one team (possibly neither). The only other requirement is due to a unique feature of mooball: an infinitely long net, which must be placed as either a horizontal or vertical line in the plane at a non-integer coordinate, such as $x = 0.5$. FJ must pick teams so that it is possible to separate the teams by a net. The cows are unwilling to move to make this true.

Help a farmer out! Compute for Farmer John the number of ways to pick a red team and a blue team satisfying the above requirements, modulo $10^9 + 7$.

INPUT FORMAT (input arrives from the terminal / stdin):

The first line of input contains a single integer N.

The next N lines of input each contain two space-separated integers x_i and y_i. It is guaranteed that the x_i form a permutation of $1 \dots N$, and same for the y_i.

OUTPUT FORMAT (print output to the terminal / stdout):

A single integer denoting the number of ways to pick a red team and a blue team satisfying the above requirements, modulo $10^9 + 7$.

SAMPLE INPUT:

```
2
1 2
2 1
```

SAMPLE OUTPUT:

```
2
```

We can either choose the red team to be cow 1 and the blue team to be cow 2, or the other way around. In either case, we can separate the two teams by a net (for example, $x = 1.5$).

SAMPLE INPUT:

```
3
1 1
2 2
3 3
```

SAMPLE OUTPUT:

```
10
```

Here are all ten possible ways to place the cows on teams; the ith character denotes the team of the ith cow, or . if the ith cow is not on a team.

```
RRB
R.B
RB.
RBB

.RB
.BR
BRR
BR.
B.R
BBR
```

SAMPLE INPUT:

```
3
1 1
2 3
3 2
```

SAMPLE OUTPUT:

```
12
```

Here are all twelve possible ways to place the cows on teams:

```
RRB
R.B
RBR
RB.
RBB
.RB
.BR
BRR
BR.
BRB
B.R
BBR
```

SAMPLE INPUT:

```
40
1 1
2 2
3 3
4 4
5 5
6 6
7 7
8 8
9 9
10 10
11 11
12 12
13 13
14 14
15 15
16 16
17 17
18 18
19 19
20 20
21 21
22 22
23 23
24 24
25 25
26 26
27 27
28 28
29 29
30 30
31 31
32 32
33 33
34 34
35 35
36 36
37 37
38 38
39 39
40 40
```

SAMPLE OUTPUT:

```
441563023
```

Make sure to output the answer modulo $10^9 + 7$.

SCORING:

- Input 5: $N \leq 10$
- Inputs 6-9: $N < 200$

- Inputs 10-13: $N \leq 3000$
- Inputs 14-24: No additional constraints.

Problem credits: Dhruv Rohatgi

Contest has ended. No further submissions allowed.