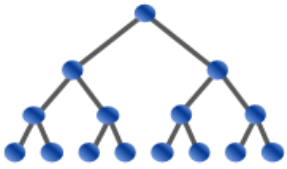

USACO 2024 JANUARY CONTEST, PLATINUM
PROBLEM 2. MERGING CELLS

Return to Problem List

Contest has ended.

****Note: The memory limit for this problem is 512MB, twice the default.****

Bessie is having fun playing a famous online game, where there are a bunch of cells of different labels and sizes. Cells get eaten by other cells until only one winner remains.

There are N ($2 \leq N \leq 5000$) cells in a row labeled $1 \dots N$ from left to right, with initial sizes $s_1, s_2, \dots, s_N$ ($1 \leq s_i \leq 10^5$). While there is more than one cell, a pair of adjacent cells is selected uniformly at random and merged into a single new cell according to the following rule:

If a cell with label a and current size c_a is merged with a cell with label b and current size c_b, the resulting cell has size $c_a + c_b$ and label equal to that of the larger cell, breaking ties by larger label. Formally, the label of the resulting cell is $\begin{cases} a & c_a > c_b \\ b & c_a < c_b \\ \max(a,b) & c_a = c_b \end{cases}$.

For each label i in the range $1 \dots N$, the probability that the final cell has label i can be expressed in the form $\frac{a_i}{b_i}$ where $b_i \not\equiv 0 \pmod{10^9+7}$. Output $a_i b_i^{-1} \pmod{10^9+7}$.

INPUT FORMAT (input arrives from the terminal / stdin):

The first line contains N.

The next line contains $s_1, s_2, \dots, s_N$.

OUTPUT FORMAT (print output to the terminal / stdout):

The probability of the final cell having label i modulo 10^9+7 for each i in $1 \dots N$ on separate lines.

SAMPLE INPUT:

```
3
1 1 1
```

SAMPLE OUTPUT:

```
0
500000004
500000004
```

There are two possibilities, where $(a,b) \to c$ means that the cells with labels a and b merge into a new cell with label c.

```
(1, 2) -> 2, (2, 3) -> 2
(2, 3) -> 3, (1, 3) -> 3
```

So with probability $1/2$ the final cell has label 2 or 3.

SAMPLE INPUT:

```
4
3 1 1 1
```

SAMPLE OUTPUT:

```
666666672
0

166666668
166666668
```

The six possibilities are as follows:

```
(1, 2) -> 1, (1, 3) -> 1, (1, 4) -> 1
(1, 2) -> 1, (2, 4) -> 4, (1, 4) -> 1
```

$(1, 2) \rightarrow 1, (3, 4) \rightarrow 4, (1, 4) \rightarrow 1$
$(2, 3) \rightarrow 3, (1, 3) \rightarrow 1, (1, 4) \rightarrow 1$
$(2, 3) \rightarrow 3, (3, 4) \rightarrow 3, (1, 3) \rightarrow 3$
$(3, 4) \rightarrow 4, (2, 4) \rightarrow 4, (1, 4) \rightarrow 4$
$(3, 4) \rightarrow 4, (1, 2) \rightarrow 1, (1, 4) \rightarrow 1$

So with probability $2/3$ the final cell has label 1, and with probability $1/6$ the final cell has label 3 or 4.

SCORING:

- Input 3: $N \leq 8$
- Inputs 4-8: $N \leq 100$
- Inputs 9-14: $N \leq 500$
- Inputs 15-22: No additional constraints.

Problem credits: Benjamin Qi

Contest has ended. No further submissions allowed.